

Introducing my fall 1987 course on Mathematical Methodology

The purpose of this introduction is three-fold:

- (0) to give you some idea of what to expect from this course,
- (1) to tell you how I propose to reach my goals, and
- (2) to explain what will be expected from you.

The idea of giving this course has been born out of dissatisfaction with the ways in which mathematics has usually been taught since I was a student. The courses I followed consisted of facts, facts, and more facts: the triple (definition, theorem, proof) repeated ad infinitum. Very useful if you wish to apply or extend the theory presented and totally useless otherwise. Later I saw a different type of mathematics course emerge: tricks, tricks, and more tricks, recipes for solving special types of problems and endless exercises so as to make their execution a routine job. Useful perhaps when that routine job is what you are heading for, but, again, totally useless otherwise. Furthermore, particularly in countries where rhetoric is confused with effective use of language, we have seen courses that are the cult of pure form: the students are only taught a pompous language and mathematics is reduced to mannerisms.

The students that, like the wild animal being prepared for its tricks in the circus called "life", expects only training as sketched above, will be severely disappointed: by his standards he will learn next to nothing.

Let me quote, by way of contrast and for inspiration, what G.R.Elton wrote about the teaching of history:

"Three or four years spent at a university cannot teach a man to know history; they cannot train him as a politician or publicist or publisher; they can at best begin to lay some foundation for a view of the world and (universities being what they are) are likely to lay foundations which, as later experience shows, need to be broken up. None of this invites blame: the impossible need not be attempted. But if those years do not produce an effective conditioning of the reasoning mind, if they do not teach a man to think better than otherwise he would have done, they may justly be condemned as a waste of time."

Well, I would be grieved to hear if your years at UT could, in Elton's words, "justly be condemned as a waste of time". In the name of "an effective conditioning of the reasoning mind", in order to teach you to "think better than otherwise you would have done", this course is about the doing of mathematics. It is my purpose to bring into the public domain what in traditional mathematics courses is left implicit.

It is not my purpose to turn, in a one-semester course, all of you into as many excellent mathematicians. Remember: the impossible need not be attempted. But I can and shall try to give you the opportunity of becoming a better mathematician than otherwise you would have been and propose to do so by exposing at least the major components of the mathematical craft and by warning you explicitly for some of the more common mistakes that the experienced competent mathematician avoids out of habit.

How to raise such a course above the level of motherhood statements --the level that is characteristic for the self-improvement or management courses sold by the training industry-- has been told to us by William Blake (1757 - 1827):

"He who would do good to another must do it in minute particulars
 General Good is the plea of the scoundrel, hypocrite and flatterer
 For Art and Science cannot exist but in minutely organized particulars."

The moral of this quotation is that, in principle, no detail will be too small for our attention. I also like this quotation for the way in which it mentions Art and Science in the same breath, for, indeed, the analogy between the performing artist and the productive scientist is not so far-fetched at all. Among the very basic things the young pianist has to learn is how to take care of his finger nails: we shall encounter similar care to be taken, so to speak, of our mathematical finger nails. Also, the performing artist must become very familiar with the potential and the limitation of his instruments; so must we with our instruments, such as pen, paper, and the printed page.

The analogy goes further. Just as a professor at a conservatory represents a musical style (to the extent that it is often possible to identify the master by listening to his pupils), I represent a mathematical style. It is up to you to decide later to what degree to adopt and to improve it. One thing, however, you are not allowed to do, viz. to reject it offhand for the sole reason that it does not reflect the way of doing mathematics you are used to. Of course it doesn't! That is precisely why you are here. This whole course is no more and no less than an invitation to take the experiment of changing some of your reasoning habits and adopting some new modes of expression. As you take the experiment you will notice that it is not acquiring the new habits that presents the greatest problem, for that is getting rid of the old ones. Perfecting oneself is as much unlearning as it is learning.

This is obvious. Yet it needs to be stressed because stupid educationists have invented for the educational process the profoundly inadequate term "knowledge transfer", suggesting a uni-directional stream of knowledge towards an accumulating recipient, who becomes monotonically more knowledgeable. This is a caricature at best.

In general, I would like to reduce the waste of time and effort. I shall not give you the magic formula that will protect you for the rest of your life from exploring dead ends, but should be able to make you alert for the most obvious blind alleys in which the inexperienced problem solver wastes most of his time. I should be able to teach you how to stay away from complicated solutions when a simple one exists. I should be able to make you aware of some of the techniques for designing a crisp argument.

In order to create that awareness, it does not suffice to solve problems or to prove theorems: we have to discuss and compare solutions and proofs. It does not suffice to identify after the fact a blind alley, we have to discuss whether it has some clear characteristics of the dead end and whether we could have seen them beforehand. It does not suffice to say "I like that argument better than that other one." but we must be able to say why we prefer the one argument over the other and must understand the justification for our preference. In the course of the process we should never forget that we are scientists, and that we have the obligation to raise our preferences above the level of personal tastes to choices that have a technical justification.

I can see three ways of doing so. I can present the alternatives, tell you my preferences, and explain their justification. The next way requires a

little bit more participation from your side: I provide the alternatives and together we discuss their merits. In the third pattern you provide the alternatives as well. It is the last pattern I prefer most. My target is to leave you at the end of each lecture with a clear question, about which you are supposed to think before the next lecture. Usually different students come at the next lecture with very different answers; and those different answers provide the material for comparison.

This last format implies a multiple obligation from your side. Between lectures you should try to solve the problem I gave you and to think about how you would present your solution. The next lecture you should be willing to come to the blackboard and present your solution to all of us. And finally you must accept that your solution and its presentation is discussed. If I may use the musical analogy again, I would like to conduct part of this course as a "master class" of, say, a teacher of pianists. These could be called "public piano lessons". In such performances you should participate in a non-competitive spirit, eager to learn how what you did can be improved. The ability of discerning high quality unavoidably implies the ability of identifying shortcomings.

(I mentioned the non-competitive spirit explicitly, because these days, excellence is a fashionable concept. But excellence is a competitive notion, and that is not what we are heading for: we are heading for perfection.)

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I hope that I have made it clear that I expect from you active participation. Yours is the role of wetstones and not of sponges; please remember that because when I am addressing sponges I always run dry very quickly. There is one specific occasion in which you must open your mouth immediately: you have to ask for clarification when you don't know what I am talking about. The technical problem of a course like this is that you are of highly varied backgrounds, with which I am not familiar. When illustrating something with an example I always run the risk of using a term some of you don't know. If so, let me know: interrupt me immediately so that I can supply the missing information.

Let me close this introduction with a final advice, just in case you develop the lurking suspicion that I am giving this course for my own fun. Of course I hope to give this course for my own fun, but remember that I fervently hope to give it for your fun as well. There should be no such thing as boring mathematics.

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