Simplifying a proof in our book

In [0], pp. 66-69, we show how the conditional distribution of $\wedge$ over $\forall$ can be derived from the one-point rule and the other axioms. I dragged sets into the picture, for which misbehaviour I apologize; here is a simpler argument. Representing "the non-empty range" for the dummy x by $r.x \vee [x=y]$ we show

$$[\langle\forall x: r.x \vee [x=y]: t.x \wedge Q\rangle \equiv \langle\forall x: r.x \vee [x=y]: t.x\rangle \wedge Q]$$

To this end we observe

$$\begin{aligned}
& \langle\forall x: r.x \vee [x=y]: t.x \wedge Q\rangle \\
= \quad & \{\text{splitting the term}\} \\
& \langle\forall x: r.x \vee [x=y]: t.x\rangle \wedge \langle\forall x: r.x \vee [x=y]: Q\rangle \\
= \quad & \{\text{see } (*) \text{ below}\} \\
& \langle\forall x: r.x \vee [x=y]: t.x\rangle \wedge Q \quad .
\end{aligned}$$

(*) We observe

$$\begin{aligned}
& \langle\forall x: r.x \vee [x=y]: Q\rangle \\
= \quad & \{\text{splitting the range}\} \\
& \langle\forall x: r.x: Q\rangle \wedge \langle\forall x: [x=y]: Q\rangle \\
= \quad & \{\text{one-point rule}\} \\
& \langle\forall x: r.x: Q\rangle \wedge Q \\
= \quad & \{\text{see } (**) \text{ below}\} \\
& Q \quad .
\end{aligned}$$

(**) We observe

$$\begin{aligned}
& [Q \Rightarrow \langle\forall x: r.x: Q\rangle] \\
= \quad & \{\Rightarrow \text{ distributes -like } \vee \text{- over } \forall \text{ in consequent}\}
\end{aligned}$$

$$
\begin{aligned}
& [\langle \forall x : r.x : \; Q \Rightarrow Q \rangle] \\
= & \quad \{\text{pred. calc}\} \\
& [\langle \forall x : r.x : \; true \rangle] \\
= & \quad \{\text{pred. calc., e.g. [0], p.66, (90)}\} \\
& [true] \\
= & \quad \{\text{pred. calc.}\} \\
& true \quad .
\end{aligned}
$$

[0] Edsger W. Dijkstra & Carel S. Scholten "Predicate Calculus and Program Semantics", Springer-Verlag New York Inc., 1990

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